Panel Unit Roots Tests for Cross-Sectionally Correlated Panels: A Monte Carlo Comparison

by

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Abstract

This paper deals with the finite sample performance of a set of unit root tests for cross correlated panels. As is well known, univariate tests are not powerful to reject the null of a unit root for the usual economic variables while panel tests, by exploiting the large number of cross-section units, provide a device to increase the power of unit root tests. We investigate the finite sample properties of recently proposed panel unit root tests for cross-sectionally correlated panels. Specifically, the size and power of Choi's (2002), Bai and Ng's (2003), Moon and Perron's (2003), and Phillips and Sul's (2003) tests are analyzed by a Monte Carlo simulation study. In synthesis, Moon and Perron's (2003) tests show good size and power for different values of T and N and model specifications. Focusing on Bai and Ng's (2003) procedure, the simulation study highlights first that the suggested ADF test for the nonstationary analysis of the common factor lack of power, and secondly the simulation shows that the pooled Dickey-Fuller-GLS test provides higher power than the pooled ADF test for the analysis of nonstationary properties of the idiosyncratic components. Choi's (2002) tests are strongly oversized when the common factor influences the cross-section units heterogeneously. Finally, all the tests lack power when a deterministic trend is included in the data generating process.

Key words **:** Panel unit root test, Cross section dependence, Monte Carlo Simulation

JEL classification: C2, C3, C5.

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1. Introduction

This paper analyzes the finite sample properties of a set of unit root tests for cross-correlated panels which have been proposed in the recent years. As is well known, given the short span of many macroeconomic time series, univariate unit root tests generally lack power. Panel unit root tests are a device to increase the power of unit root tests by exploiting the information included across the units. However panel unit root tests are valid instruments only when applied to a cross section of units that are not cross-correlated.

There is now a large amount of empirical literature, see for example Backus and Kehoe (1992), that provides evidence on the strong comovement between economic variables. This is why in recent years a discrete number of panel unit root tests have been proposed which are well suited for analyzing cross-correlated panels. Many of these consider models where the cross-section units are generated by a linear dynamic factor model where the common and idiosyncratic components influence all the units in the panel. In these models the common factor plays an important role: it allows the dimensions of the cross sectional covariance of the units to be reduced. Thus, when computing panel unit root tests, all the procedures propose methodologies that permit the use of de-factored, i.e. independent, data.

In the paper we analyze the finite sample properties of four recently proposed tests, specifically the Choi (2002), Bai and Ng (2003), Moon and Perron (2003) and Phillips and Sul (2003) tests. Although these tests utilize different procedures to detect the common and idiosyncratic components, their properties can be analyzed in a common environments in order to compare the finite performance of the tests for different values of T, N and different specifications.

In synthesis, our Monte Carlo study highlights that Moon and Perron's (2003) tests show good size and power for different values of T and N and model specifications. Focusing on Bai and Ng's (2003) procedure, the simulation study highlights first that the suggested ADF test for the nonstationary analysis of the common factor lacks power, and secondly the simulation shows that the pooled Dickey-Fuller-GLS test provides higher power than the pooled ADF test when analyzing the nonstationary properties of the idiosyncratic components. Choi's (2002) tests are strongly oversized when the common factor influences the cross-section units heterogeneously. Finally, we find that all the tests lack power when a deterministic trend is included in the process.

In Section 2 we briefly review the panel unit root tests analyzed in the paper. Section 3 presents the Monte Carlo simulation study and Section 4 the conclusions.

2. A brief review of recently proposed panel unit root tests for cross-correlated panels.

2.1 The Moon and Perron (2003), (MP) procedure

We can start the analysis considering the following dynamic panel model

$$\begin{aligned} y_{it} &= a_{i0} + x_{it} \\ x_{it} &= r_i x_{it-1} + u_{it} \\ u_{it} &= b_i' f_t + e_{it} \end{aligned} \tag{1}$$

where the observed $y_{it}\left(i=1,\ldots,N;t=1,\ldots,T\right)$ are generated by a deterministic component a_{i0} and an autoregressive process x_{it}. The error component u_{it} follows a factor model where the common factor component $f_t = C\left(L\right)v_t$ is generated by a stationary linear process with each matrix of polynomial lag *C(L)* of dimension (*K*,*K*) with $v_t \sim iid\left(0, I_K\right)$ and b_i are the factor loading coefficients. The idiosyncratic shocks e_{it} follows a linear process $e_{it} = D_i\left(L\right)e_{it}$, with $e_{it} \sim iid\left(0, s_{ei}^2\right)$. The model assumes that b_i, n_t, e_{it} are mutually independent.

The null hypothesis in model (1) is

$$H_0:\ r_i = 1 \text{ for all } i = 1,\ldots,N \tag{2}$$

against the alternative of

$$H_A:\ \left|r_i\right| < 1 \text{ for some } i. \tag{3}$$

It is simple to observe from (1) that under the null hypothesis of $r_i = 1$, y_{it} is influenced by two components: the integrated factors $\sum_{s=1}^{T} f_s$ and the integrated idiosyncratic errors $\sum_{s=1}^{T} e_{is}$. While the model allows for both integrated or cointegrated factors, i.e. in this case the rank of $C\left(1\right)$ is *r*<*K*, the model excludes the possibility of cointegrating relations of the integrated idiosyncratic errors.

MP propose removing cross sectional dependence in (1) by multiplying the observed matrix *Y* of dimension (*TxN*) by the projection matrix Q_b and computing the unbiased pooled autoregressive estimator as

$$r_{\text{pool}}^{+} = \frac{tr\left(Y_{-1} Q_b Y'\right) - NT\, l_e^N}{tr\left(Y_{-1} Q_b Y'_{-1}\right)}, \tag{4}$$

where Y_{-1} the matrix of lagged observed data, $tr(\cdot)$ the trace operator and l_e^N is the cross-sectional average of the one-sided long run variance of the idiosyncratic errors e_{it}.

In order to obtain feasible statistics, MP procedure requires estimating the number K of factors in (1), and the projection matrix Q_b. The first task is accomplished by using a methodology similar to that proposed in Bai and Ng (2002), while the vector (or matrix when K>1) of factor loading $\hat{b}$ and the connected projection matrix $Q_{\hat{b}}$ are obtained by estimating the principal components of $\hat{e}'\hat{e} = \left(Y - \hat{r}_{\text{pool}} Y_{-1}\right)' \left(Y - \hat{r}_{\text{pool}} Y_{-1}\right)$, where $\hat{r}_{\text{pool}}$ is the OLS pooled autoregressive estimate.

To test the null hypothesis (2), MP suggest two test statistics

$$t_a^* = \frac{\sqrt{N}T\left(\hat{r}_{\text{pool}}^+ - 1\right)}{\sqrt{\dfrac{2\hat{f}_e^4}{w_e^4}}} \tag{5a}$$

$$t_b^* = \sqrt{N}T\left(\hat{r}_{\text{pool}}^+ - 1\right)\sqrt{\frac{1}{NT^2} tr\left(Y_{-1} Q_b Y_{-1}'\right) \frac{\hat{w}_e^2}{f_e^4}} \tag{5b}$$

where $\hat{r}_{\text{pool}}^+$ is the bias-corrected pooled autoregressive estimate of (4) and $\hat{w}_{\hat{e}}^2$ and $\hat{f}_{\hat{e}}^4$ are respectively the estimates of the cross sectional average of long run variance of $\hat{e}_{it}$ and the cross sectional average of $w_{\hat{e},i}^4$.

Under the null H_0, MP show that for $(N,T \to \infty)$ with $N/T \to 0$ the statistics (5a) and (5b) have a standard normal distribution.

2.2 The Bai and Ng (2003) (BN) procedure

Bai and Ng (2003) propose a different methodology to test for panel unit root. To analyze their strategy in the context of model (1), let us assume that the null hypothesis (2) holds and (1) admits only one factor. In this case we have that

$$y_{it} = a_{i0} + b_i \sum_{t=1}^{T} f_t + \sum_{t=1}^{T} e_{it} .$$

Both the common factor and the idiosyncratic components are non stationary. In synthesis, BN suggest testing the common factor and the idiosyncratic components for a unit root separately. BN show that a consistent estimate of the (differenced) common factor Δf_t, and the associated vector of factor loadings b, can be obtained by computing the principal components of the (differenced) matrix of observed data *Y*. The (differenced) estimates of residuals are thus computed as $\Delta\hat{e}_{it} = \Delta y_{it} - \hat{b}_i^{'}\Delta\hat{f}_t$ while by re-cumulating $\hat{f}_t = \sum_{s=2}^{T}\Delta f_s$ and $\hat{e}_{it} = \sum_{s=2}^{T}\Delta\hat{e}_{is}$, $i = 1,\dots,N$.

It is simple now to test for the null hypothesis of a unit root the common factor $\hat{f}_t$ and each idiosyncratic components $\hat{e}_{it}$ separately. For the first component and when only a factor is detected[1], BN suggest using the ADF test, while a modified version of Stock and Watson (1988) common trend tests have to be used when more than one factor is detected. For the idiosyncratic components BN propose a method based on meta-analysis and presented originally in Maddala and Wu (1999) and in Choi (2001). This method consists of combining the p-values of the ADF test computed for each idiosyncratic errors $\hat{e}_{it}$. The statistic is given by

$$P_{\hat{e}}^{C} = \frac{-2\sum_{i=1}^{N}\log p_{\hat{e}i}^{c} - 2N}{\sqrt{4N}} \xrightarrow{d} N(0,1) \tag{6}$$

where $p_{\hat{e}i}^{c}$ is the p-value of the ADF test on the estimated residual $\hat{e}_{it}$. The statistic (6) converges for $(N,T \to \infty)$ to a standard normal distribution. As we will see later, the Dickey-Fuller-GLS proposed in Elliott et al. (1996) can be fruitfully used.

2.3 The Phillips and Sul (2003) (PS) procedure

The PS method is similar to the previously analyzed Moon and Perron (2003) procedure. It differs in that only one factor is permitted in (1) and f_t is independently distributed as $N(0,1)$ across time. Instead of using principal components, PS suggest computing the previously defined projection matrix Q_b by a moment-based method. The de-factored data are used to compute a series of panel unit root tests. The first one is defined as

$$G_{OLS}^{++} = \frac{1}{\sqrt{N}s_x}\sum_{i=1}^{N-1}\left[\frac{\hat{r}_i^{+} - 1}{\hat{s}_{\hat{r}^{+}}} - m_x\right] \xrightarrow{d} N(0,1) \tag{7}$$

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[1] The number of factors is estimated by using Bai and Ng's (2002) procedure.

where $\hat{r}_i^+$ and $\hat{s}_{\hat{r}^+}$ are respectively the cross-sectional autoregressive estimates and its standard errors computed for each i computed from the de-factored data. [2] The parameters m_x and s_x are the mean and standard error of statistic. PS show that for $(N, T \to \infty)$ (7) converges to a standard normal distribution.

As in Bai and Ng (2003), PS also propose meta-analysis tests. Specifically the test which seems to work best is given by

$$Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N-1} \Phi^{-1}\left(p_{\hat{e}i}^c\right), \tag{8}$$

where $p_{\hat{e}i}^c$ is , as before, the p-value of the ADF test associated with cross section element i, and $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function for a standard normal variable. Expression (8) converges to a standard normal distribution. Note finally that both tests (7) and (8) require summing up only N-1 elements because the PS procedure reduces the cross sectional dimension by 1.

2.4 The Choi (2002) procedure

The Choi (2002) procedure utilizes a two-way error-component model which differs from (1) mainly because each cross section i is influenced homogeneously by the single factor f_t, or, in other words, Choi's (2002) procedure requires that $b_i = 1$ for all i.

In order to remove the common component, the procedure requires first demeaning the data, following the method suggested in Elliott et al. (1996), and second subtracting from the demeaned data the cross-sectional means. It is simple to show that in this case the new variables are independent across the units i for large N and T.

Thus, as before, meta-analysis can be fruitfully used in order to obtain panel unit root test. Choi (2002) combines p-values from Dickey-Fuller-GLS tests computed for each unit i. The three suggested pooled tests are

$$P = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \log\left(p_{\hat{e}i}^c + 1\right) \tag{9a}$$

$$Z = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}\left(p_{\hat{e}i}^c\right) \tag{9b}$$

[2] Phillips and Sul (2002) also propose the G_{EMS}^{++} test based on the median estimates of r_i^+ which seems to have marginal better property than the G_{OLS}^{++} test.

$$L = \frac{1}{\sqrt{p^2 N/3}} \sum_{i=1}^{N} \log\left(\frac{p_{\hat{e}i}^c}{1 - p_{\hat{e}i}^c} \right). \quad (9c)$$

Note that the P test is a modification of Fisher's (1932) inverse chi-square tests and rejects the null hypothesis for positive large value of the statistics. The test statistic (9b) has been previously analyzed and the L is a logit test. These last two tests reject the null for large negative values of the statistics. Finally the P, Z and L tests under the null converge to a standard normal distribution as $(N, T \to \infty)$.

3. The Monte Carlo simulation study

In this section we present an extensive Monte Carlo simulation study to compare the finite sample properties of the tests reviewed in the previous section.

As a data generating process we use a simple modification of model (1):

$$\begin{aligned} z_{it} &= a_{i0} + z_{it}^0 \\ z_{it}^0 &= z_{it-1}^0 + t \sum\nolimits_{j=1}^{K} b_{ij} f_{jt} + \sqrt{K} e_{it}. \end{aligned} \quad (10)$$

The simulation results are based on 1000 iterations at T =50, 100, 200,400; and N=20, 40, 60, 80. Nominal size for the simulation results was set at 0.05. Initially, we permit for the presence of only one factor, i.e. K=1, but we also study the tests properties for K=3.

The t parameter assumes the values of $t = 0,1$, and 4. When $t = 0$ corresponds to the hypothesis of absence of cross-sectional dependence, while for $t = 1$ the same importance is given to the common factor and idiosyncratic components. Finally when $t = 4$ greater importance is attributed to the common factor components.

We try three different specifications of factor loading vector b. In the first specification we set $b_{ij} \sim N(0,1)$. In the second, we assume that $b_{ij} \sim U[1,4]$ and this implies high cross sectional dependence. We also study the tests properties when $b_{ij} = 1$, i.e. when the cross-section units i are influenced by the common factor homogenously. We assume also that $(f_{jt}, e_{it}, a_{i0}) \sim iid\ N(0, I_3)$.

3.1 Results

In Table 1, we present the computed size of tests. Note that in the case of Bai and Ng's (2003) tests, we also include the pooled Dickey-Fuller-GLS test proposed by Elliot et al. (1996). To save space only the values for *N*=20, 80 have been included in Table 1. [3]

When $t = 1$ and $b_{ij} \sim N(0,1)$, and for large *T* and *N*, all the tests seem to have good size. The only exception are Choi's (2002) tests which are strongly oversized. The reasons are probably connected to the imposed hypothesis of the non-homogeneous effects of the common factor on the cross section units. It is simple to show that when the effects are heterogeneous Choi's (2002) procedure, cross-sectional dependence cannot be removed from the data. Thus, if cross-section units are heterogeneously influenced by the common factor, then there is a greater chance of Choi's (2002) tests erroneously rejecting the null hypothesis of a unit root in the panel.

When $t = 0$, only Phillips and Sul's (2003) tests seem to suffer from oversize, especially for small *N* and T, while for $t = 4$, Choi's (2002) tests show a stronger oversize when compared to $t = 1$. There is the same problem for Moon and Perron's (2003) tests, especially for small *N*. Bai and Ng's (2003) tests seem only marginally influenced by the stronger variability of the common component, but as in the case of MP's tests, the distortion is strongest when the value of *N* is small. This is probably connected to the difficulty of estimating the common variation consistently when only a small number of units are included in the data generating process.

In Table 1 the size of the tests are presented also when $t = 1$ and $b_{ij} \sim U[1,4]$. All tests show a strong distortion of size especially for small values of *N* and *T*. Finally in Table 1 we present the size of tests when *K*=3, i.e. three common factors are included in (10), and when $b_{ij} = 1$. In the former case, Phillips and Sul's (2003) tests, which can take into account only one factor, are largely oversized. Some distortion is noted also for the Moon and Perron (2003) t_a^* test.

As expected, the Choi (2003) tests show the correct size when $b_{ij} = 1$. Interestingly, the size of the other panel unit root tests is not strongly influenced by this hypothesis.

In Table 2 we report the size-adjusted power of the test statistics. The power is computed assuming that $r_i \sim U[0.98,1]$. Thus the average $\bar{r}$=0.99. Summarizing from Table 2, it emerges that the power of all tests rises for larger values of *T* and *N*.

When compared with the other tests, Moon and Perron's (2003) t_a^* and t_b^* tests generally show the highest power for all *N*, *T* and for different specifications. Focusing on Bai and Ng's (2003) tests, the power of the ADF test related to the common factor component is not different

[3] The values for *N*=40,60 are freely available upon request from the author.

from size [4], especially for small T and N. The power of the pooled Dickey-Fuller-GLS tests is always higher than the power of the pooled ADF tests. Unlike Phillips and Sul's (2003) results, the G_{OLS}^{++} test seems to have higher power than the Z test. The power of Choi's (2002) tests is similar to that of the pooled Bai and Ng (2003) and Phillips and Sul (2003) tests, with the exception of $b_{ij} = 1$ where the power of Choi's (2002) tests is higher than that of the latter tests.

Two points must be mentioned before concluding. In the simulation study we hypothesize that the true number of factors is known. When this number has to be estimated, the size and power of the Moon and Perron (2003) and Bai and Ng (2003) tests can be influenced. However, Bai and Ng (2002) show that with at least 20 cross-sections, i.e. our minimum value in the simulation study, the number of factors can be precisely estimated. Thus our results are probably only marginally influenced by this problem. Secondly, we conducted some simulation studies, not reported for brevity, including a deterministic trend in (10). While generally all tests seem to have a good size, the power of tests is dramatically low. As a result, researchers must be careful when using these panel unit root tests for variables that include a deterministic trend in the process.

4. Conclusion

In the paper we analyze the finite sample properties of some panel unit root tests when the cross-sectional units are correlated. From the analysis some useful results emerge. First, when only a deterministic constant is included in the model, the Moon and Perron (2003) tests show a good size and power for different specifications and different values of T and N. Second, while the Bai and Ng's (2003) pooled tests on the null hypothesis that idiosyncratic components are non-stationary have good size and power, especially when the Dickey-Fuller-GLS test is used, the ADF test used to analyze the nonstationary properties of the common component has low power, generally not different from the size. Third, Choi's (2002) tests are largely oversized, except when the cross-section units respond homogeneously to the common factor. Finally, unlike Phillips and Sul's (2003) results, the G_{OLS}^{++} test seems to show better properties than the pooled Z test, but large size distortions are detected for both tests when more than one factor is included in the simulation study. Finally we find all tests lack power when a deterministic trend is included in the process. Thus, researchers must be careful when using the previously reviewed panel unit root tests with

[4] A higher power is noted when $r_i \sim U[0.8,1]$. For example for T=400, N=40, $t = 1$ and $b_{ij} \sim N(0,1)$ the power of ADF test is 0.83.

variables, such as real GNP or industrial production, that are probably influenced by a deterministic trend.

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Table 1. Size of Tests

$$DGP:\begin{cases} z_{it}=a_{i0}+z_{it}^{0} \\ z_{it}^{0}=z_{it-1}^{0}+t\sum_{j=1}^{K}b_{ij}f_{jt}+\sqrt{K}e_{it} \\ a_{i0},f_{jt},e_{it}\sim iid\ N(0,1) \end{cases}$$

	T	N	Bai and Ng (2003)			Moon, Perron (2003)		Choi (2002)			Phillips, Sul (2003)	
			Common Factor(s) ADF or MQ(*) test	$P_{\hat{e}}^{C}$ Pooled ADF	$P_{\hat{e}}^{C}$ Pooled DF-GLS	t_a^*	t_b^*	P	Z	L	G_{OLS}^{++}	Z
K=1, $t=1$, $b_{ij}\sim N(0,1)$	50	20	0,031	0,034	0,023	0,029	0,060	0,129	0,140	0,138	0,051	0,040
	50	80	0,048	0,025	0,007	0,013	0,047	0,230	0,304	0,294	0,066	0,021
	100	20	0,056	0,059	0,057	0,047	0,051	0,129	0,137	0,132	0,059	0,048
	100	80	0,036	0,033	0,034	0,030	0,042	0,251	0,310	0,297	0,063	0,037
	200	20	0,045	0,067	0,068	0,074	0,070	0,123	0,140	0,133	0,064	0,054
	200	80	0,040	0,059	0,049	0,038	0,041	0,255	0,328	0,314	0,052	0,036
	400	20	0,041	0,067	0,072	0,084	0,064	0,108	0,119	0,111	0,061	0,046
	400	80	0,039	0,060	0,054	0,054	0,050	0,233	0,313	0,294	0,047	0,049
K=1, $t=0$, $b_{ij}\sim N(0,1)$	50	20	0,031	0,058	0,023	0,038	0,069	0,058	0,046	0,045	0,063	0,067
	50	80	0,042	0,025	0,007	0,018	0,054	0,039	0,039	0,042	0,335	0,263
	100	20	0,042	0,057	0,059	0,055	0,061	0,073	0,063	0,059	0,072	0,067
	100	80	0,048	0,037	0,029	0,033	0,057	0,065	0,057	0,056	0,190	0,162
	200	20	0,043	0,072	0,063	0,076	0,071	0,066	0,056	0,060	0,070	0,069
	200	80	0,041	0,053	0,048	0,047	0,048	0,058	0,049	0,048	0,093	0,091
	400	20	0,045	0,075	0,056	0,078	0,054	0,061	0,044	0,048	0,058	0,060
	400	80	0,044	0,061	0,058	0,051	0,052	0,053	0,044	0,041	0,073	0,071
K=1, $t=4$, $b_{ij}\sim N(0,1)$	50	20	0,029	0,034	0,027	0,056	0,062	0,241	0,312	0,296	0,082	0,028
	50	80	0,052	0,022	0,007	0,029	0,043	0,318	0,427	0,419	0,144	0,026
	100	20	0,056	0,062	0,059	0,122	0,092	0,232	0,306	0,287	0,086	0,052
	100	80	0,034	0,035	0,034	0,066	0,074	0,318	0,445	0,438	0,127	0,042
	200	20	0,046	0,068	0,069	0,141	0,102	0,238	0,314	0,300	0,075	0,049
	200	80	0,038	0,057	0,051	0,064	0,057	0,355	0,457	0,453	0,078	0,042
	400	20	0,041	0,071	0,077	0,143	0,101	0,213	0,296	0,289	0,070	0,053
	400	80	0,041	0,061	0,053	0,094	0,072	0,326	0,450	0,443	0,054	0,050
K=1, $t=1$, $b_{ij}\sim U[1,4]$	50	20	0,033	0,037	0,021	0,029	0,048	0,127	0,129	0,127	0,082	0,027
	50	80	0,038	0,023	0,011	0,023	0,039	0,215	0,262	0,260	0,133	0,031
	100	20	0,044	0,054	0,064	0,099	0,091	0,142	0,158	0,153	0,075	0,040
	100	80	0,044	0,036	0,044	0,046	0,066	0,263	0,335	0,333	0,129	0,040
	200	20	0,040	0,073	0,072	0,114	0,090	0,120	0,147	0,143	0,069	0,046
	200	80	0,045	0,049	0,064	0,066	0,074	0,240	0,309	0,298	0,074	0,046
	400	20	0,051	0,065	0,076	0,112	0,088	0,111	0,116	0,120	0,071	0,057
	400	80	0,048	0,053	0,053	0,063	0,056	0,221	0,301	0,291	0,048	0,045
K=1, $t=1$, $b_{ij}=1$	50	20	0,045	0,044	0,028	0,033	0,064	0,050	0,044	0,042	0,073	0,029
	50	80	0,042	0,025	0,003	0,017	0,041	0,030	0,033	0,035	0,054	0,023
	100	20	0,035	0,061	0,057	0,060	0,065	0,082	0,062	0,062	0,061	0,051
	100	80	0,037	0,038	0,029	0,028	0,042	0,064	0,045	0,044	0,066	0,030
	200	20	0,034	0,074	0,066	0,075	0,061	0,081	0,076	0,073	0,084	0,080
	200	80	0,043	0,066	0,056	0,046	0,051	0,051	0,046	0,044	0,061	0,050
	400	20	0,039	0,063	0,057	0,081	0,059	0,045	0,043	0,042	0,063	0,058
	400	80	0,046	0,054	0,054	0,046	0,041	0,051	0,041	0,041	0,063	0,051
K=3, $t=1$, $b_{ij}\sim N(0,1)$	50	20	0,011	0,050	0,027	0,032	0,066	0,104	0,101	0,098	0,092	0,071
	50	80	0,019	0,028	0,011	0,007	0,042	0,147	0,193	0,196	0,225	0,118
	100	20	0,043	0,057	0,054	0,068	0,077	0,091	0,095	0,093	0,120	0,079
	100	80	0,051	0,041	0,036	0,029	0,052	0,183	0,227	0,230	0,279	0,133
	200	20	0,060	0,077	0,062	0,080	0,068	0,088	0,105	0,102	0,108	0,063
	200	80	0,059	0,043	0,047	0,055	0,059	0,192	0,244	0,244	0,255	0,155
	400	20	0,070	0,067	0,054	0,102	0,078	0,088	0,087	0,085	0,117	0,095
	400	80	0,064	0,052	0,065	0,057	0,054	0,150	0,207	0,209	0,238	0,164

(*) For K>1, Bai and Ng's (2003) common trend MQ_c^c test

Table 2. Power of Tests

$$DGP:\begin{cases} z_{it} = a_{i0} + z_{it}^{0} \\ z_{it}^{0} = r_i z_{it-1}^{0} + t\sum_{j=1}^{K} b_{ij} f_{jt} + \sqrt{K} e_{it} \\ a_{i0}, f_{jt}, e_{it} \sim iid\ N(0,1) \\ r_i \sim U[0.98,1] \end{cases}$$

	T	N	Bai and Ng (2003) Common Factor(s) ADF or MQ(*) test	Bai and Ng (2003) $P_{\hat{e}}^{C}$ Pooled ADF	Bai and Ng (2003) $P_{\hat{e}}^{C}$ Pooled DF-GLS	Moon, Perron (2003) t_a^*	Moon, Perron (2003) t_b^*	Choi (2002) P	Choi (2002) Z	Choi (2002) L	Phillips, Sul (2003) G_{OLS}^{++}	Phillips, Sul (2003) Z
$K=1$, $t=1$, $b_{ij} \sim N(0,1)$	50	20	0,065	0,071	0,097	0,535	0,478	0,090	0,122	0,119	0,108	0,074
	50	80	0,053	0,081	0,184	0,889	0,865	0,130	0,176	0,176	0,136	0,118
	100	20	0,034	0,084	0,221	0,878	0,863	0,231	0,306	0,281	0,183	0,126
	100	80	0,056	0,190	0,584	0,983	0,977	0,283	0,365	0,347	0,363	0,262
	200	20	0,073	0,161	0,539	0,927	0,920	0,479	0,623	0,581	0,362	0,236
	200	80	0,074	0,438	0,978	0,998	0,999	0,688	0,783	0,780	0,819	0,696
	400	20	0,094	0,606	0,967	0,984	0,983	0,897	0,943	0,942	0,811	0,695
	400	80	0,102	0,932	1,000	0,997	0,996	0,962	0,969	0,968	0,983	0,922
$K=1$, $t=0$, $b_{ij} \sim N(0,1)$	50	20	0,055	0,061	0,110	0,328	0,303	0,116	0,166	0,164	0,103	0,079
	50	80	0,046	0,069	0,184	0,919	0,893	0,308	0,428	0,413	0,105	0,054
	100	20	0,058	0,095	0,237	0,865	0,855	0,279	0,458	0,428	0,215	0,135
	100	80	0,050	0,206	0,653	0,999	0,999	0,723	0,921	0,907	0,328	0,213
	200	20	0,069	0,181	0,504	0,885	0,876	0,569	0,742	0,736	0,352	0,234
	200	80	0,073	0,526	0,999	1,000	1,000	0,999	1,000	1,000	0,898	0,686
	400	20	0,112	0,715	0,997	0,995	0,993	0,996	0,999	0,999	0,889	0,732
	400	80	0,110	0,984	1,000	1,000	1,000	1,000	1,000	1,000	1,000	0,989
$K=1$, $t=4$, $b_{ij} \sim N(0,1)$	50	20	0,077	0,068	0,068	0,322	0,287	0,067	0,066	0,063	0,069	0,080
	50	80	0,052	0,054	0,081	0,490	0,441	0,062	0,072	0,067	0,075	0,082
	100	20	0,027	0,050	0,125	0,445	0,503	0,101	0,112	0,104	0,089	0,083
	100	80	0,062	0,112	0,233	0,519	0,516	0,094	0,102	0,101	0,146	0,164
	200	20	0,067	0,077	0,240	0,540	0,535	0,164	0,185	0,176	0,120	0,112
	200	80	0,066	0,189	0,529	0,618	0,622	0,162	0,211	0,197	0,241	0,293
	400	20	0,093	0,222	0,538	0,662	0,662	0,427	0,473	0,454	0,224	0,215
	400	80	0,112	0,342	0,755	0,608	0,616	0,405	0,453	0,435	0,287	0,302
$K=1$, $t=1$, $b_{ij} \sim U[1,4]$	50	20	0,051	0,071	0,076	0,377	0,329	0,083	0,109	0,095	0,074	0,063
	50	80	0,054	0,081	0,111	0,647	0,637	0,114	0,145	0,147	0,114	0,100
	100	20	0,049	0,073	0,093	0,376	0,414	0,134	0,138	0,137	0,090	0,088
	100	80	0,050	0,078	0,158	0,623	0,604	0,148	0,200	0,193	0,122	0,147
	200	20	0,062	0,092	0,198	0,568	0,565	0,259	0,343	0,338	0,115	0,126
	200	80	0,066	0,217	0,553	0,759	0,758	0,413	0,520	0,488	0,354	0,319
	400	20	0,086	0,103	0,245	0,479	0,476	0,328	0,403	0,397	0,113	0,111
	400	80	0,099	0,521	0,836	0,846	0,848	0,744	0,816	0,798	0,656	0,567
$K=1$, $t=1$, $b_{ij}=1$	50	20	0,051	0,060	0,082	0,344	0,301	0,100	0,132	0,132	0,074	0,081
	50	80	0,048	0,096	0,193	0,942	0,930	0,319	0,437	0,431	0,202	0,120
	100	20	0,059	0,065	0,164	0,500	0,489	0,174	0,222	0,209	0,120	0.086
	100	80	0,062	0,167	0,470	0,964	0,963	0,572	0,766	0,751	0,328	0,259
	200	20	0,072	0,163	0,585	0,918	0,907	0,571	0,729	0,698	0,304	0,197
	200	80	0,081	0,286	0,875	0,991	0,990	0,917	0,971	0,969	0,671	0,491
	400	20	0,106	0,425	0,894	0,958	0,955	0,936	0,970	0,968	0,652	0,469
	400	80	0,074	0,855	0,998	0,999	0,990	0,998	0,998	0,998	0,964	0,900
$K=3$, $t=1$, $b_{ij} \sim N(0,1)$	50	20	0,056	0,068	0,089	0,531	0,470	0,124	0,157	0,151	0,109	0,089
	50	80	0,046	0,097	0,165	0,917	0,897	0,157	0,208	0,201	0,106	0,101
	100	20	0,049	0,076	0,188	0,724	0,672	0,219	0,281	0,280	0,174	0,113
	100	80	0,063	0,160	0,520	0,997	0,997	0,367	0,483	0,477	0,301	0,151
	200	20	0,062	0,159	0,504	0,911	0,896	0,548	0,686	0,671	0,323	0,255
	200	80	0,074	0,415	0,971	1,000	1,000	0,786	0,850	0,846	0,652	0,382
	400	20	0,105	0,342	0,796	0,835	0,820	0,901	0,939	0,931	0,564	0,402
	400	80	0,092	0,960	1,000	1,000	1,000	1,000	0,999	0,999	0,953	0,801

(*) For K>1, Bai and Ng's (2003) common trend MQ_c^c test